ATTACHMENT F

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May 4, 1994

Calibration Of MODIS PC HgCdTe Channels

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Calibration Algorithms For PC HgCdTe Channels

LMS Quadratic Fit To Calibration Data

$$V_Q = A + B\, L_{EFF} + C\, L_{EFF}^2$$

V_Q = signal voltage

L_{EFF} = effective radiance (includes background)

$$L_{EFF} = \tau_0\, L_S + (1+\delta)^2\, \tau_{CO}\, (1-\tau_{WO})\, L_B$$

L_S = scene (signal) radiance

L_B = background Planck blackbody radiance

τ_{CO} = cold optics transmission

τ_{WO} = warm optics transmission

τ_0 = optical system transmission

$$1 + \delta = \frac{(f/)_S}{(f/)_B}$$

$(f/)_S$ = optics port (signal) f-number

$(f/)_B$ = effective background f-number

If the image quality does not change then A, B, and C are independent of the instrument temperature.

Derivation Of Expression For Effective Radiance

Voltage out of detector circuit depends on the irradiance at the detector. For a DC coupled circuit the voltage V across the load is

$$V = a + b(E_S+E_B) + c(E_S+E_B)^2$$

E_S=irradiance due to scene (signal)

E_B=irradiance due to background

$$E_S = \frac{\pi\ \tau_0\ L_S}{4(f/)_S^2}$$

$$E_B = \frac{\pi\ \tau_{CO}\ \epsilon_B\ L_B}{4(f/)_B^2}$$

ϵ_B=effective emissivity of background

Since cold optics radiance is insignificant for MODIS

$$\epsilon_B = 1 - \tau_{WO}$$

Let
$$1+\delta = \frac{(f/)_S}{(f/)_B}$$

$$E_B = \frac{\pi(1+\delta)^2\ \tau_{CO}(1-\tau_{WO})L_B}{4(f/)_S^2}$$

$$E_S+E_B = \frac{\pi}{4(f/)_S^2}\left[\tau_0\ L_S+(1+\delta)^2\ \tau_{CO}\ (1-\tau_{WO})\ L_B\right] = \frac{\pi\ L_{EFF}}{4(f/)_S^2}$$

$$L_{EFF} = \tau_0\ L_S+(1+\delta)^2\ \tau_{CO}\ (1-\tau_{WO})\ L_B\ , \qquad \text{Q.E.D.}$$

Measurement Of Scene Radiance From Orbit

Use of the inverse calibration equation is convenient for orbital data.

$$L_{EFF} = x_0 + m\,x + q\,x^2$$

x=digital number (counts)

Note that x_0, m, and q are independent of instrument temperature.

Since $\tau_0 L_S = L_{EFF} - (1+\delta)^2 \tau_{CO} (1-\tau_{WO}) L_B$

$$L_S = (x_0 + m\,x + q\,x^2 - \kappa L_B)/\tau_0$$

$$\kappa = (1+\delta)^2 (1-\tau_{WO})\,\tau_{CO}$$

If the additional warm surfaces seen by the detector (beyond the optical port) radiate as blackbodies then κ must be replaced by

$$\kappa = \left[(1+\delta)^2 - \tau_{WO})\right] \tau_{CO}$$

Since $\tau_0 = \tau_{CO}\tau_{WO}$, either τ_0 or τ_{WO} must be measured as a function of instrument temperature.

On-Board Calibration

From the values of scene signal x_S and space signal x_{SP} the scene radiance L_S can be found without using the value of x_0.

$$L_S = \left[m + q(x_S + x_{SP})\right](x_S - x_{SP})/\tau_0$$

Using the on-board blackbody radiance L_{BB} and associated signal x_{BB} the slope m can be found. Since q is a much slower varying function of detector responsivity than m, it can be considered constant (for a constant detector temperature).

$$m = \left[\tau_0 L_{BB} - q(x_{BB} + x_{SP})\right]/(x_{BB} - x_{SP})$$

Using this value of m the new value of x_0 becomes

$$x_0 = \tau_0 L_{BB} - m\,x_{BB} - q\,x^2_{BB} + \kappa\,L_{BB}$$

Non-Linearity

Let L_M=maximum effective radiance in the instrument calibration with the corresponding voltage V_M.

Define the reference line to lie be between the points L=0 (V=A) and L=L_M on the LMS quadratic fit of the calibration data. Since

$$V_M = A + B\,L_M + C\,L_M^2$$

the slope of this reference line is $B+CL_M$. The equation of the reference line is

$$V_L = A + (B+CL_M)L_{EFF}$$

At $L_{EFF}=L_M/2$ the difference between the voltage values of the quadratic fit (V_Q) and the reference line (V_L) is

$$V_Q-V_L = -CL_M^2/4$$

The non-linearity with respect to the reference line is

$$(NL)_{REF} = |(V_Q-V_L)/(V_L-A)| \quad \text{at } L_{EFF}=L_M/2$$

$$= \left| \frac{CL_M^2/4}{(B+CL_M)L_M/2} \right|$$

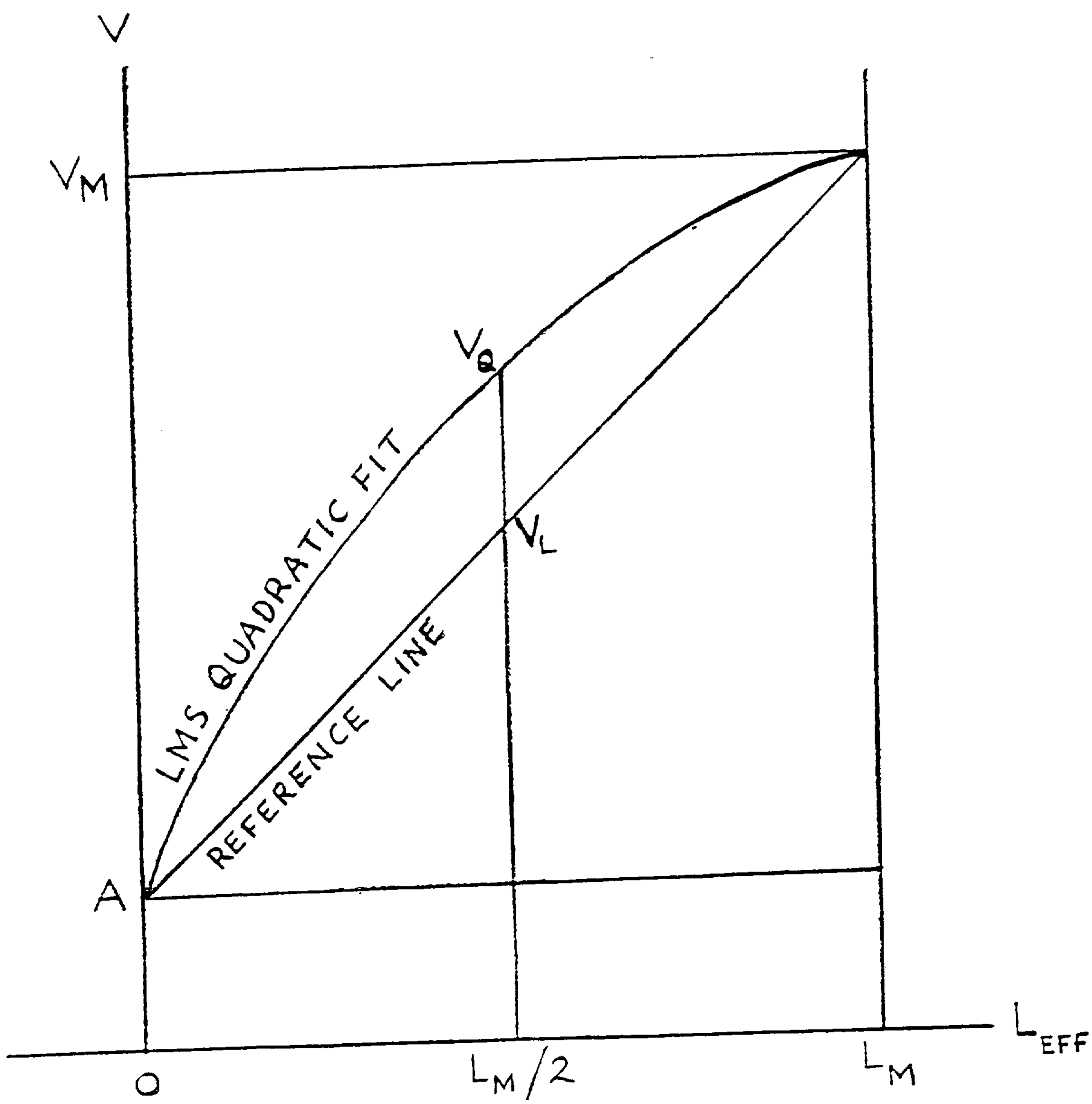
$$= \left| \frac{CL_M/2}{B+CL_M} \right|$$

Since the system non-linearity $(NL)_S$ is approximately half this value

$$(NL)_S = \left| \frac{z/4}{1+z} \right|$$

$$z = CL_M/B$$

LMS QUADRATIC FIT TO CALIBRATION DATA



$$(NL)_{REF} = \frac{V_Q - V_L}{V_L - A}$$

$$(NL)_S = 0.5(NL)_{REF}$$